0.1 Single Phase AC circuit

0.1.1 AC circuit with a pure resistance:

Consider an electrical circuit consists of a pure resistance R with an alternating voltage $V = V_m sin\omega t$ as shown in the Figure 19. The current i flowing in the circuit is expressed as

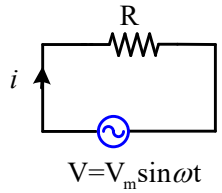


Figure 1

$$i = \frac{V_m sin\omega t}{R} = I_m sin\omega t$$

where $I_m = \frac{V_m}{R}$

The instantaneous power consumed by the resistance R in the above circuit is

$$\begin{aligned} P &= vi = (V_m sin\omega t)(I_m sin\omega t) \\ &= V_m I_m sin^2(\omega t) \\ &= V_m I_m \frac{1 - cos2\omega t}{2} \\ &= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} cos2\omega t \end{aligned}$$

The equation consists of two terms. The first term is called as the constant power term. The second term is consists of $\frac{V_m I_m}{2} cos2\omega t$ which is periodically varying with frequency 2ω, twice the input frequency. The average power over a period of time is zero.

$$\begin{aligned} P &= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} cos2\omega t \\ P_{av} &= \int_0^{2\pi} \left(\frac{V_m I_m}{2} - \frac{V_m I_m}{2} cos2\omega t \right) \\ &= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \int_0^{2\pi} \frac{1}{2\pi} (cos2\omega t)\, d\omega t \\ &= \frac{V_m I_m}{2} - 0 \\ &= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \\ &= VI \end{aligned}$$

The details of the input alternating voltage v current i and power p waveforms are as shown in Figure 2

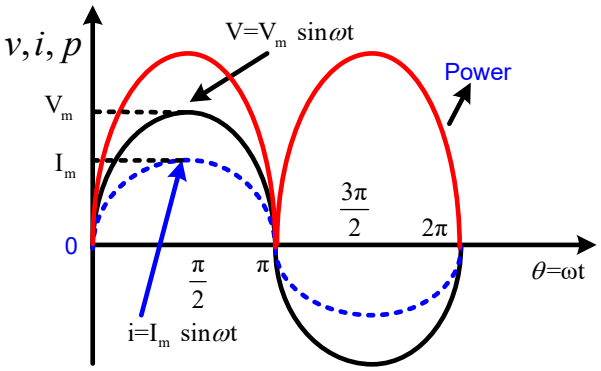


Figure 2

0.1.2 AC circuit with a pure Inductance:

Consider an electrical circuit consists of a inductor L with an alternating voltage $V = V_m sin\omega t$ as shown in the Figure 3. The current i flowing in the circuit is expressed as

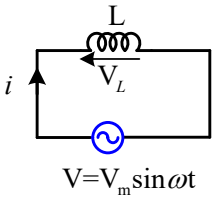


Figure 3

$$V_L = -L\frac{dl}{di}$$

$$\begin{aligned} dl &= \frac{V_L}{L} dt = \frac{1}{L} V_m sin\omega t\, dt \\ i &= \frac{V_m}{L} \int sin\omega t\, dt \\ &= \frac{V_m}{\omega L} (-cos\omega t) \\ &= \frac{V_m}{X_L} sin(\omega t - \frac{\pi}{2}) \\ &= I_m sin\left(\omega t - \frac{\pi}{2}\right) \end{aligned}$$

The instantaneous power consumed by the inductance L in the above circuit is

$$\begin{aligned} P &= vi = (V_m sin\omega t) I_m sin\left(\omega t - \frac{\pi}{2}\right) \\ &= V_m I_m sin(\omega t)(-cos\omega t) \\ &= -\frac{1}{2} V_m I_m sin2\omega t \end{aligned}$$

The power consumed by the inductance consists of $\frac{V_m I_m}{2} cos2\omega t$ which is periodically varying with frequency 2ω, twice the input frequency.

The details of the input alternating voltage v current i and power p waveforms are as shown in Figure 4

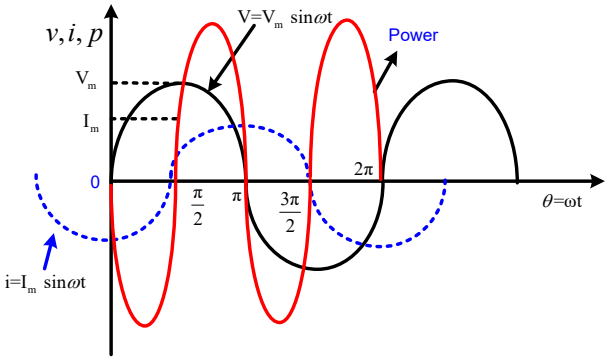


Figure 4

0.1.3 AC circuit with a pure Capacitance:

Consider an electrical circuit consists of a capacitor C with an alternating voltage $V = V_m sin\omega t$ as shown in the Figure 3. The current i flowing in the circuit is expressed as

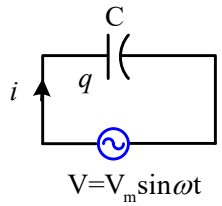


Figure 5

$$\begin{aligned} i &= \frac{dq}{dt} = \frac{dCv}{dt} \\ &= C\frac{d}{dt}V_m sin\omega t \\ &= \omega C V_m cos\omega t \\ &= \frac{V_m}{1/\omega C} sin(\omega t + \pi/2) \\ &= \frac{V_m}{X_C} sin(\omega t + \pi/2) \\ &= I_m sin(\omega t + \pi/2) \end{aligned}$$

where $I_m = \frac{V_m}{X_C}$, $X_C = \frac{V_m}{\omega C}$

The instantaneous power consumed by the inductance L in the above circuit is

$$\begin{aligned} P &= vi = (V_m sin\omega t) I_m sin\left(\omega t - \frac{\pi}{2}\right) \\ &= V_m I_m sin(\omega t)(-cos\omega t) \\ &= -\frac{1}{2} V_m I_m sin2\omega t \end{aligned}$$

The instantaneous power is.

$$\begin{aligned} P &= vi = (V_m sin\omega t) I_m sin\left(\omega t + \frac{\pi}{2}\right) \\ &= V_m I_m sin\omega t cos\omega \\ &= \frac{1}{2} V_m I_m sin2\omega t \end{aligned}$$

The details of the input alternating voltage v current i and power p waveforms are as shown in Figure 6

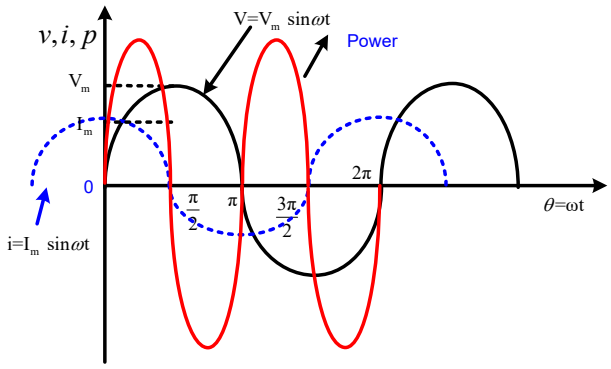


Figure 6

0.1.4 Series R-L circuit:

Consider an electrical circuit consists of a resistor R and inductor L connected in series with an alternating voltage $V = V_m sin\omega t$ as shown in the Figure 7. The voltage across the resistor is V_R and across the inductor is V_L.

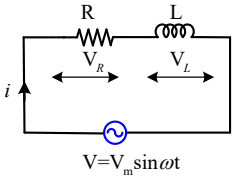


Figure 7

The current flowing in the network I, the voltage across resistor V_R is in phase with I and the voltage across inductor V_L leads the current by 90°. The phasor diagram of I V_R are V_L as shown in Figure 12.

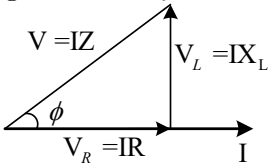


Figure 8

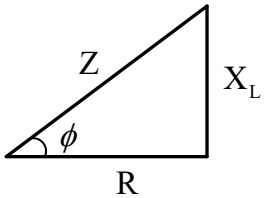


Figure 9

From the phasor diagram, the the resultant voltage V can be expressed as

$$\begin{aligned} V &= \sqrt{V_R^2 + V_L^2} \\ &= \sqrt{(IR)^2 + (IX_L)^2} \\ &= I\sqrt{R^2 + X_L^2} \\ &= I\sqrt{R^2 + X_L^2} \\ &= IZ \end{aligned}$$

where Z is

$$Z = \sqrt{R^2 + X_L^2}$$

The phase angle Φ between the voltage and and the current is

$$\Phi = tan^{-1}\frac{X_L}{R}$$

The instantaneous power consumed by the circuit is

$$\begin{aligned} P &= vi = (V_m sin\omega t) I_m sin\,(\omega t - \phi) \\ &= \frac{1}{2}V_m I_m [cos\phi - cos(2\omega t - \phi) \\ &= \frac{1}{2}V_m I_m cos\phi - \frac{1}{2}V_m I_m cos(2\omega t - \phi) \end{aligned}$$

The second term is periodically varying quantity and its frequency is twice the applied frequency. Average of the power is zero. The first term is called as the constant power term which represents the power consumed in the circuit.

$$\begin{aligned} P &= \frac{1}{2}V_m I_m cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}cos\phi \\ &= VIcos\phi \end{aligned}$$

The details of the input alternating voltage v current i and power p waveforms are as shown in Figure 13

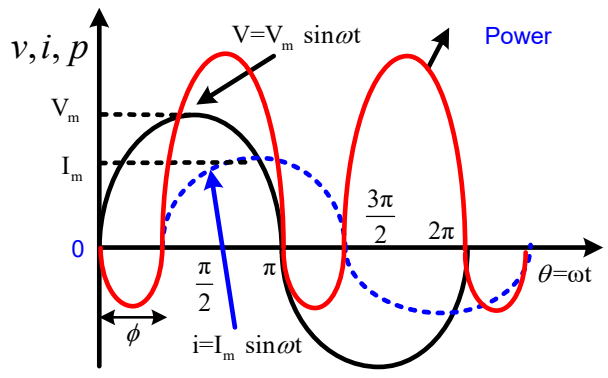


Figure 10

0.1.5 Series R-C circuit:

Consider an electrical circuit consists of a resistor R and capacitor C are connected in series with an alternating voltage $V = V_m sin\omega t$ as shown in the Figure 7. The voltage across the resistor is V_R and across the capacitor is V_C.

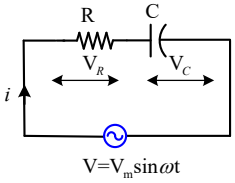


Figure 11

The current flowing in the network I, the voltage across resistor V_R is in phase with I and the voltage across capacitor V_C lags the current by 90°. The phasor diagram of I, V_R and V_L are as shown in Figure **??** (a).

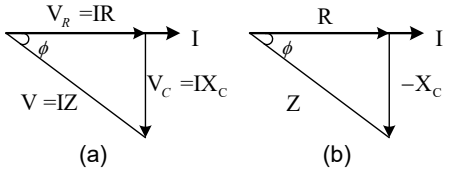


Figure 12

From the phasor diagram, the the resultant voltage V can be expressed as

$$\begin{aligned} V &= \sqrt{V_R^2 + V_C^2} \\ &= \sqrt{(IR)^2 + (-IX_C)^2} \\ &= I\sqrt{R^2 + X_C^2} \\ &= I\sqrt{R^2 + X_C^2} \\ &= IZ \end{aligned}$$

where Z is

$$Z = \sqrt{R^2 + X_C^2}$$

The phase angle Φ between the voltage and and the current is

$$\Phi = tan^{-1}\frac{X_C}{R}$$

The instantaneous power consumed by the circuit is

$$\begin{aligned} P &= vi = (V_m sin\omega t) I_m sin\,(\omega t + \phi) \\ &= \frac{1}{2}V_m I_m [cos\phi - cos(2\omega t - \phi) \\ &= \frac{1}{2}V_m I_m cos\phi - \frac{1}{2}V_m I_m cos(2\omega t - \phi) \end{aligned}$$

The second term is periodically varying quantity and its frequency is twice the applied frequency. Average of the power is zero. The first term is

called as the constant power term which represents the power consumed in the circuit.

$$\begin{aligned} P &= \frac{1}{2}V_m I_m cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}cos\phi \\ &= VIcos\phi \end{aligned}$$

The details of the input alternating voltage v current i and power p waveforms are as shown in Figure 13

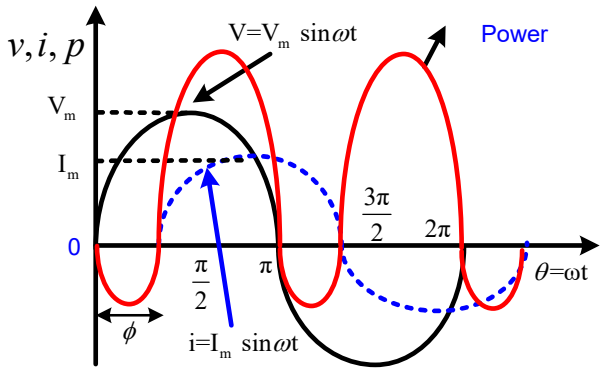


Figure 13

0.2 Problems on AC circuits

Q1) Two parallel circuits comprising of (i) a coil of resistance of 20Ω and inductance of 0.07 H and (ii) a resistance of 50 Ω in series with a condenser of capacitance 60 μF are connected across 230 V, 50 Hz. Calculate the main current and power factor of the arrangement.

Solution:

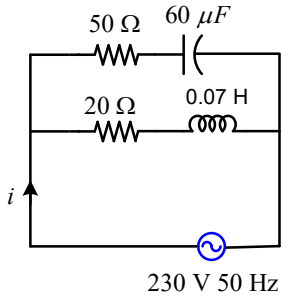


Figure 14

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.07 = 22\ \Omega$$
$$Z_1 = 20 + j22 = 29.7\angle 47.7^\circ$$

$$I_1 = \frac{230}{29.7\angle 47.7^\circ} = 7.74\angle -47.7^\circ$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}}$$
$$= 53\ \Omega$$
$$Z_2 = 50 - j53 = 72.86\angle -46.66^\circ$$

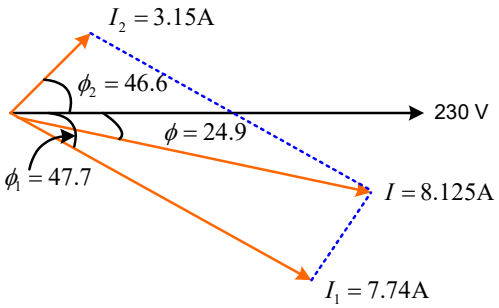
$$I_2 = \frac{230}{72.86\angle -46.66^\circ} = 3.156\angle 46.6^\circ$$

$$Z = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$
$$= \frac{(20 + j22)(50 - j53)}{20 + j22 + 50 - j53}$$
$$= 25.657 + j11.93$$

$$I = \frac{230}{25.657 + j11.93} = 8.125\angle -24.92^\circ$$

Power factor is

$$p.f = cos(\phi) = cos(\angle -24.92^\circ) = 0.907$$



Q2) Two impedances $Z_1 = 150 - j157\ \Omega$ and $Z_2 = 100 - j110\ \Omega$ Find (i) Branch Currents (ii) Total current (iii) Total power (iv) Draw vector Diagram

Solution:

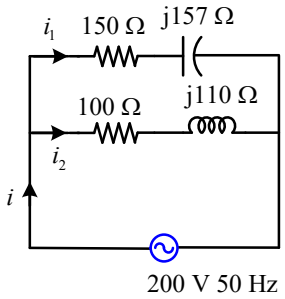


Figure 15

$$Z_1 = 150 - j157 = 217\angle -46.3^\circ$$
$$I_1 = \frac{200}{217\angle -46.3^\circ} = 0.921\angle 46.3^\circ$$

$$Z_2 = 100 + j110 = 148.66\angle 47.75^\circ$$
$$I_2 = \frac{200}{148.66\angle 47.75^\circ} = 1.345\angle -47.75^\circ$$

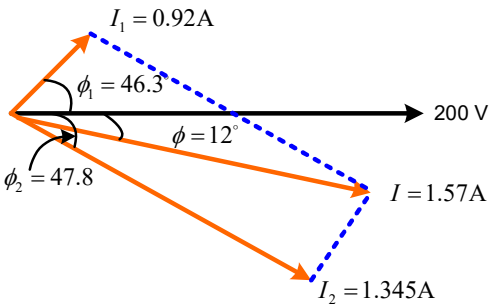
$$Z = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$
$$= \frac{(150 - j157)(100 + j110)}{150 - j157 + 100 + j110}$$
$$= 124 + j26.53$$

$$I = \frac{200}{124 + j26.53} = 1.577\angle -12^\circ$$

(iii) Total power

$$Power = VIcos(\phi) = 200 \times 1.577cos(-12^\circ)$$
$$= 308.6Watts$$

(iv) Draw vector Diagram



Q3) In the arrangement shown in the figure. Calculate the impedance between AB and the phase angle between voltage and current. Also calculate the total power consumed, if the applied voltage between AB is $200\angle 30°$.

Solution:

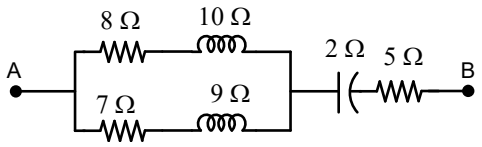


Figure 16

$$\begin{aligned} Z_1 &= 8 + j10 = 12.8\angle 51.34° \\ Z_1 &= 7 + j9 = 11.4\angle 52.12° \end{aligned}$$

Two Impedances are in parallel

$$\begin{aligned} Z_3 &= \frac{(8+j10)(7+j9)}{8+j10+7+j9} \\ &= 3.733 + j4.73 \end{aligned}$$

The impedance between AB is

$$\begin{aligned} Z_{AB} &= 3.733 + j4.73 + 5 - j2 \\ &= 8.733 + j2.73 = 9.147\angle 17.36° \end{aligned}$$

$$I = \frac{200\angle 30°}{9.147\angle 17.36°} = 21.865\angle 12.64°$$

Phase angle between voltage and current $\angle 12.64°$

The total power consumed

$$\begin{aligned} Power &= VIcos(\phi) \\ 200 \times 21.865 cos(\angle 12.64°) & \\ &= 4267 Watts \end{aligned}$$

2019-Jan 3 b) A resistance of 7 is connected in series with a pure inductance of 31.8 mH and the circuit is connected to a 100 V 50 Hz sinusoidal supply. Calculate i) circuit current ii) Phase angle iii) Power factor iv) Power.

Solution:

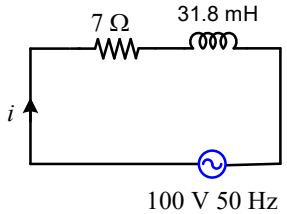


Figure 17

$$\begin{aligned} R &= 7 \\ X_L &= 2\pi fL = 2\pi \times 50 \times 31.8 \times 10^{-3} \\ &= 10\ \Omega \\ Z &= 7 + j10 = 12.2\angle 55° \end{aligned}$$

i) circuit current

$$\begin{aligned} I &= \frac{V}{Z} = \frac{100}{12.2\angle 55°} \\ &= 8.19\angle -55° \end{aligned}$$

ii) Phase angle

$$\begin{aligned} \phi &= tan^{-1}\frac{X_L}{R} = tan^{-1}\frac{10}{7} \\ &= 55° \end{aligned}$$

iii) Power factor

$$cos\phi = cos(55°) = 0.573$$

iv) Power

$$\begin{aligned} &= VIcos\phi = 100 \times 8.19 \times cos(55°) \\ &= 469.75w \end{aligned}$$

2019-Jan 4 c) A coil having a resistance of 20 Ω and inductance of 0.0382 H, is connected in parallel with a circuit consisting of a 150 μ F capacitor in series with 10 Ω resistor. The arrangement is connected to a 230 V, 50 Hz supply. Determine current in each branch. Also find total supply current.

Solution:

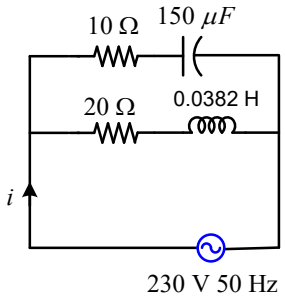


Figure 18

$$\begin{aligned} X_L &= 2\pi fL = 2\pi \times 50 \times 0.0382 \\ &= 12\ \Omega \\ Z_1 &= 20 + j12 = 23.3\angle 30.96° \end{aligned}$$

$$\begin{aligned} I_1 &= \frac{V}{Z_1} = \frac{230}{23.3\angle 30.96°} \\ &= 9.87\angle -30.96° A \end{aligned}$$

$$\begin{aligned} X_C &= \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}} \\ &= 21.22\ \Omega \\ Z_2 &= 10 + j21.22 = 23.45\angle 64.76^\circ \end{aligned}$$

$$\begin{aligned} I_2 &= \frac{V}{Z_1} = \frac{230}{23.45\angle 64.76^\circ} \\ &= 9.8\angle -64.76^\circ A \end{aligned}$$

2019-DEC 3 b) Given $V = 200sin377$ volts and $i = 8sin(377t - 30^\circ)$ Amps for an AC circuit, determine i) the power factor ii)True power iii) Apparent power iv) Reactive power indicate the unit of the power calculated.

Solution:

$$\begin{aligned} Z &= \frac{E}{I} = \frac{377}{8\angle -30^\circ} \\ &= 47.125\angle 30^\circ \\ &= 40.8 - j23.56 \end{aligned}$$

i) the power factor

$$\begin{aligned} p.f &= cos(\phi) = cos(-30^\circ) \\ &= 0.866 \end{aligned}$$

OR

$$\begin{aligned} p.f &= \frac{R}{Z} = \frac{R}{Z} = \frac{40.8}{47.125} \\ &= 0.866 \end{aligned}$$

ii)True power Actual or True Power

$$\begin{aligned} P &= VIcos\phi = 200 \times 8 \times cos(-30^\circ) \\ &= 1385.6\ Watts \end{aligned}$$

iii) Apparent power

$$\begin{aligned} P &= VI = 200 \times 8 \\ &= 1600\ (volt\ amp) \end{aligned}$$

iv) Reactive power

$$\begin{aligned} P &= VIsin\phi = 200 \times 8 \times sin(-30^\circ) \\ &= 800\ (VAR) \end{aligned}$$

2019-June 4 b) An alternating voltage of (160+j120) V is applied to a circuit and the current is given by (6+j8)A. Find the values of circuit elements by assuming f=50Hz. Calculate the power factor of the circuit and power consumed by the circuit.

Solution:

$$\begin{aligned} Z &= \frac{E}{I} = \frac{160 + j120}{6 + j8} \\ &= \frac{200\angle 36.87^\circ}{10\angle 53.13^\circ} \\ &= 20\angle -16.26^\circ \\ &= 19.2 - j5.6 \end{aligned}$$

$$R = 19.2\Omega \quad X_C = 5.6\Omega$$

Power factor of the circuit

$$\begin{aligned} p.f &= cos(\phi) = cos(-16.26^\circ) \\ &= 0.96 \end{aligned}$$

OR

$$\begin{aligned} p.f &= \frac{R}{Z} = \frac{R}{Z} = \frac{19.2}{20} \\ &= 0.96 \end{aligned}$$

Power consumed by the circuit

$$\begin{aligned} Power &= VIcos\phi = 200 \times 10 \times cos(-16.26^\circ) \\ &= 1920\ W \end{aligned}$$

OR

$$\begin{aligned} Power &= I^2R = 10^2 \times 19.2 \\ &= 1920\ W \end{aligned}$$

2017-June (15ELE15) 5 b) An alternating voltage of (80+j60) V is applied to a circuit and the current flowing through it is (-4+j10)A. Find the i) impedance of the circuit ii) phase angle iii) pf of the circuit iv) power consumed by the circuit.

Solution:

i) The impedance of the circuit

$$\begin{aligned} Z &= \frac{E}{I} = \frac{80 + j60}{-4 + j10} \\ &= \frac{100\angle 36.87^\circ}{10.77\angle 111.8^\circ} \\ &= 9.28\angle -74.93^\circ \\ &= 2.41 - j8.96 \end{aligned}$$

ii) phase angle is 74.93° Lagging

iii) Power factor of the circuit

$$\begin{aligned} p.f &= cos(\phi) = cos(-74.93^\circ) \\ &= 0.26 \end{aligned}$$

OR

$$\begin{aligned} p.f &= \frac{R}{Z} = \frac{R}{Z} = \frac{2.41}{9.28} \\ &= 0.26 \end{aligned}$$

iv)Power consumed by the circuit

$$\begin{aligned} Power &= I^2R = 10.77^2 \times 2.41 \\ &= 279.5\ W \end{aligned}$$

2019-June (17ELE15) 5 c) A series circuit of resistance of 10Ω, an inductance of 13 mH and a capacitance of 150 μF connected in series. A supply of 100 V at 50 Hz is given to the circuit. Find the impedance, current pf and power consumed in the circuit

Solution:

i) The impedance of the circuit is

$$\begin{aligned} R &= 10 \\ X_L &= 2\pi fL = 2\pi \times 50 \times 16 \times 10^{-3} = 5.02\Omega \\ X_C &= \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}} = 21.22\Omega \end{aligned}$$

The impedance of the circuit is

$$\begin{aligned} Z &= R + j(X_L - X_c) = 10j(5.02 - 21.22) \\ &= 10 - J16.2\Omega \\ &= 19.03\angle 58.313^\circ \end{aligned}$$

Current in the circuit is

$$\begin{aligned} I &= \frac{V}{Z} = \frac{100}{19.03} \\ &= 5.255A \end{aligned}$$

iii) Power factor of the circuit

$$p.f = \frac{R}{Z} = \frac{10}{19.03} = 0.525$$

iv)Power consumed by the circuit

$$\begin{aligned} Power &= I^2R = 5.255^2 \times 10 \\ &= 276.15\ W \end{aligned}$$

2019-June (15ELE15) 5 c) For the circuit shown in Figure 19 find current in all branches. Draw vector diagram

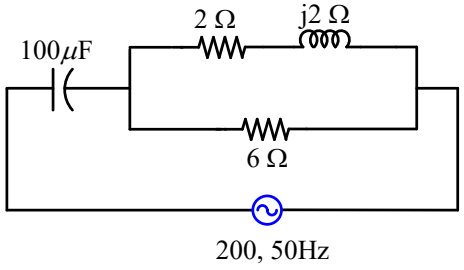


Figure 19

Solution:

i) The impedance of the circuit is

$$\begin{aligned} X_C &= \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega \end{aligned}$$

The impedance of the circuit is

$$\begin{aligned} Z &= -j31.83 + \frac{2 + j2 + 6}{(2 + j2) \times 6} \\ &= -j31.83 + 0.416 - j0.25\Omega = 0.416 - j32.08\Omega \\ &= 32.08\angle -89.25^\circ \end{aligned}$$

Current in the circuit is

$$\begin{aligned} I &= \frac{V}{Z} = \frac{200}{32.08\angle -89.25^\circ} \\ &= 6.234\angle 89.25^\circ A \end{aligned}$$

Current in (2+j2) Ω branch by branch current method

$$\begin{aligned} I_1 &= I\frac{6}{2 + j2 + 6} = 6.234\angle 89.25^\circ \frac{6}{2 + j2 + 6} \\ &= 4.536\angle 75^\circ A \end{aligned}$$

Current in 6 Ω branch by branch current method

$$\begin{aligned} I_2 &= I\frac{2 + j2}{2 + j2 + 6} = 6.234\angle 89.25^\circ \frac{2 + j2}{2 + j2 + 6} \\ &= 2.138\angle 120^\circ A \end{aligned}$$

2019-June (15ELE15) 6 c) A certain circuit takes a current of (-5+j10) amperes when applied voltage is (50+j200) volts. If the frequency of the supply is 50 Hz. i) Find the circuit elements ii) apparent power iii) reactive power iv) power factor

Solution:

$$\begin{aligned} V &= 50 + j200 = 206.15\angle 76^\circ \\ I &= -5 + j10 = 11.18\angle 116.5^\circ \end{aligned}$$

i) The impedance of the circuit is

$$\begin{aligned} Z &= \frac{V}{I} = \frac{50 + j200}{-5 + j10} \\ &= 14 - j12\Omega \\ &= 18.44\angle -40.6^\circ \end{aligned}$$

The circuit elements are

$$\begin{aligned} R &= 14\Omega \\ X_C &= 12\Omega \end{aligned}$$

iii) Apparent power

$$\begin{aligned} P &= VI = 206.15 \times 11.18 \\ &= 2304.757\ (volt\ amp) \end{aligned}$$

iv) Reactive power

$$\begin{aligned} P &= VIsin\phi = 206.15 \times 11.18 \times sin(40.6^\circ) \\ &= 1499.87\ (VAR) \end{aligned}$$

iv) the power factor

$$\begin{aligned} p.f &= cos(\phi) = cos(40.6^\circ) \\ &= 0.759 \end{aligned}$$

OR

$$\begin{aligned} p.f &= \frac{R}{Z} = \frac{R}{Z} = \frac{14}{18.44} \\ &= 0.759 \end{aligned}$$

2019-June (15ELE15) 6 c) When 220 V AC supply is applied across AB terminals for the circuit shown in Fig the power input is 3.25 kW and the current is 20 A. Find the current through Z_3

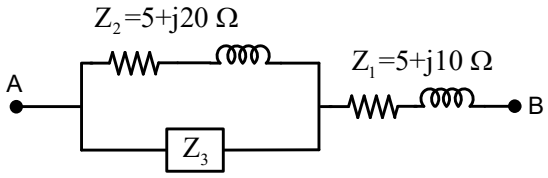


Figure 20

Solution:

$$\begin{aligned} VIcos(\phi) &= 3250 \\ cos(\phi) &= \frac{3250}{VI} = \frac{3250}{220 \times 20} \\ &= 0.7386 \\ \phi &= cos^{-1}(0.7386) \\ &= -42.38^\circ \end{aligned}$$

The current and the voltages are in phase with 42.38°, hence

$$\begin{aligned} V &= 220 \\ I &= 20\angle -42.38^\circ \end{aligned}$$

Voltage across Z_3 or $5 + j20$ is

$$\begin{aligned} &= 220\angle 0^\circ - 20\angle -42.38^\circ \times (5 + j20) \\ &= 81.13\angle 82^\circ \end{aligned}$$

The current through $5 + j20$ is I_1

$$\begin{aligned} I_1 &= \frac{81.13\angle 0^\circ}{5 + j20} \\ &= 3.93\angle -75.96^\circ \end{aligned}$$

2019-June (14ELE15) 6 a) A resistance R in series with a capacitor C is connected to 50 Hz, 240 V supply. Find the value of C so that R absorbs 300 W at 100 V. Find also maximum charge and maximum energy stored in C.

Solution:

$$\begin{aligned} &= I^2R = \frac{V^2}{R} \\ R &= \frac{V^2}{P} = \frac{100^2}{300} = 33.3\Omega \end{aligned}$$

$$I = \frac{V}{R}\frac{100}{33.3} = 3A$$

$$Z = \frac{E}{I}\frac{240}{3} = 80\Omega$$

$$\begin{aligned} Z &= R - jX_C \\ Z &= \sqrt{R^2 + X_C^2} \\ Z^2 &= R^2 + X_C^2 \\ X_C^2 &= Z^2 - R^2 = 80^2 - 33.3^2 \\ &= 5291.11 \\ X_C &= 72.74\Omega \end{aligned}$$

$$\begin{aligned} C &= \frac{1}{2 \times f \times X_C} \\ &= \frac{1}{2 \times 50 \times 72.74} \\ &= 43.8\mu F \\ V_C &= 3 \times 72.74 = 218V \end{aligned}$$

Maximum charge

$$\begin{aligned} Q &= CV_m = \\ &= 43.8 \times 10^{-6} \times 218 \times \sqrt{2} \\ &= 0.0135Coulomb \end{aligned}$$

Maximum energy stored in C

$$\begin{aligned} CV^2 &= 43.8 \times 10^{-6} \times 218^2 \\ &= 2.08J \end{aligned}$$

0.3 Three Phase Circuits

0.3.1 AC circuit with a pure resistance:

Advantages of three-phase systems :

- For power transmission, three phase transmission lines require much lesser conductor material than a single phase system.
- Three phase system is more efficient & less expensive compared to single phase system
- Three phase machine gives higher output than a single phase machine.
- Three phase motor develops uniform torque whereas single phase motor develops pulsating torque.
- Three phase system produces rotating magnetic field with stationary coils, & hence three phase induction motors are self starting.
- Voltage regulation is much better in three phase supply than single phase supply.

0.3.2 Generation of three phase power:

Three phase voltage generater is called as an alternator. The alternator consists of stator which is stationary and rotor which is rotationary part. The stator is a cylindrical in shape and has slots in its inner periphery The details of an alternator is as shown in Figure 21. The conductors are placed in the slots which are connected either in star or delta form. Rotor consists of a magnet with two poles N and S.

Three conductors AA', BB' and CC' are electrically placed by 120°. When the the rotor starts rotating in clockwise, three stator conductor cuts the magnetic flux thereby an emf is induced in the conductors. The phase of the emf generated in the coils are 120° with respect to other coils.

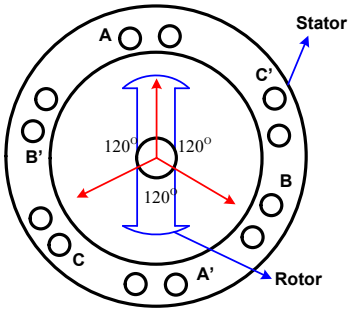


Figure 21: Three Phase generator

The voltages generated in the three coils are represented as

$$\begin{aligned} E_a &= E_m sinwt \\ E_b &= E_m sin(wt - 120) \\ E_c &= E_m sin(wt - 240) = E_m sin(wt + 120) \end{aligned}$$

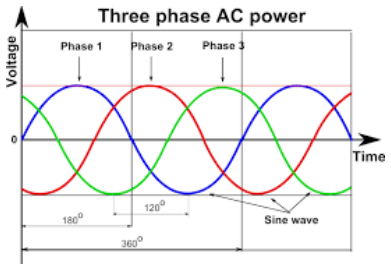


Figure 22: Phase sequence abc

Phase sequence :

The phase sequence of the three phase supply is the order in which the 3 phase voltages reach their maximum values. The phase sequence either abc' or 'acb. If the maximum values of phase voltages occur in the sequence abc then the phase of the supply voltage is abc which is as shown in Figure 23. If the maximum values of phase voltages occur in the sequence acb then the phase of the supply voltage is acb which is as shown in Figure 24.

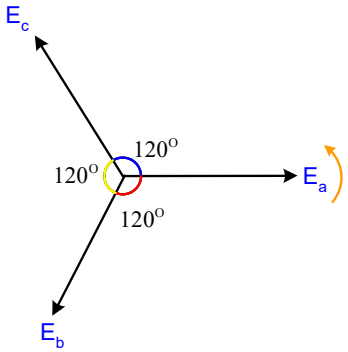


Figure 23: Phase sequence abc

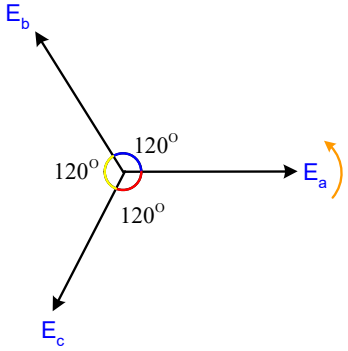


Figure 24: Phase sequence acb

The phase sequence of abc is

$$\begin{aligned} E_a &= E_m sinwt \\ E_b &= E_m sin(wt-120) \\ E_c &= E_m sin(wt-240) = E_m sin(wt+120) \end{aligned}$$

Importance of phase sequence:

- When the three phase supply with a particular sequence is given to three phase static load then the current will flow with the lines. If the phase sequence is changed then the direction of current flow will also change.
- When the three phase supply is given to the three phase induction motor, and if the phase sequence is changed then the direction of current flow will reverse and the direction of rotation of the motor also changes.

Balanced three phase supply :

- If the three phase supply is having the same magnitude and are differ by 120° with respect to other supply is called as balanced supply. The details of the balanced three phase supply is as shown in Figure **??**.
- If the three phase supply is having the different magnitude or are differ by phase angle with respect to other supply is called as unbalanced supply is as shown in Figure **??**.

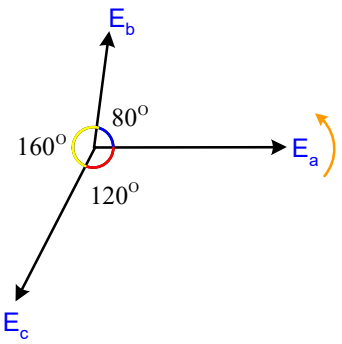


Figure 25: Phase sequence abc

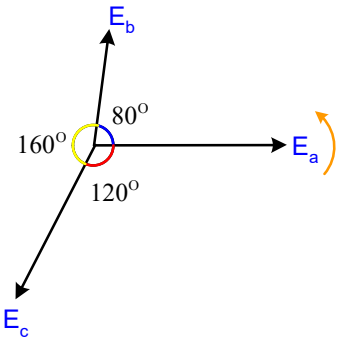


Figure 26: Phase sequence abc

Balanced Load :

- If the impedances in all the three phases are exactly equal in magnitude, then the load is said to be balanced load.
- If the three phase supply is having the different magnitude or are differ by phase angle with respect to other supply is called as unbalanced supply.

0.3.3 Three Phase Connections:

There are two types three phase load connections:

1. Star Connection (**Y**)
2. Delta Connection (Δ)

Star Connection:

The three phase star connection load as shown in Figure 27. The three phase supply voltages V_{RY}, V_{BR}, V_{YB} are connected to the three loads with a common point known as neutral N. The voltage between any two lines is called line voltage and the voltage between line and neutral point is called as phase voltage.

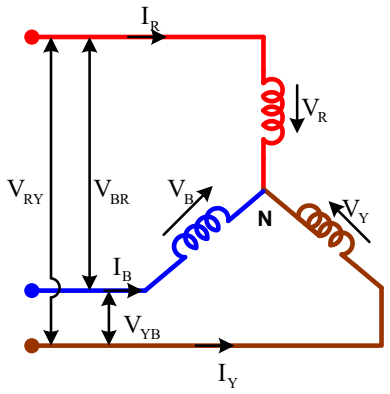


Figure 27: Star Connection

The three line voltages are:
V_{RY}, V_{BR}, V_{YB}
The three phase voltages are:
V_R, V_B, V_Y

$$\begin{aligned} V_R &= V_P \angle 0^\circ \\ V_Y &= V_P \angle -120^\circ \\ V_B &= V_P \angle 120^\circ \end{aligned}$$

$$V_{RY} = V_R - V_Y \quad (Vector\ sum)$$

From the phasor diagram which is as shown in Figure 28

$$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y cos\angle 60^\circ}$$

Let

$$V_R = V_Y = V_B = V_P$$

$$V_{RY} = \sqrt{V_P^2 + V_P^2 + 2V_P V_P \times 0.5} = \sqrt{3}V_P$$

Similarly

$$V_{YB} = \sqrt{3}V_Y$$
$$V_{BR} = \sqrt{3}V_B$$

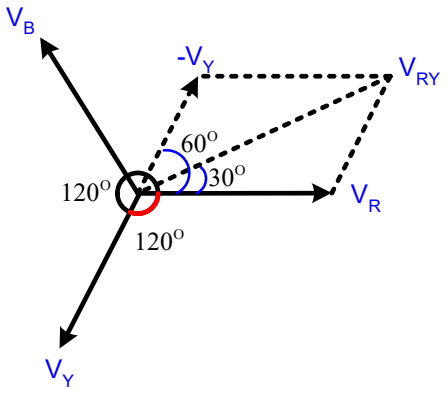


Figure 28: Phasor diagram of balanced star system

Therefore

$$V_L = \sqrt{3}V_Y = \sqrt{3}V_{PH}$$
$$I_L = I_{PH}$$

Power consumed by 3 phase circuit is

$$P = 3 \times V_{PH} \times I_{PH} cos\phi = 3\frac{V_L}{\sqrt{3}} \times I_L cos\phi = \sqrt{3}V_L I_L cos\phi$$

Delta Connection:

The three phase delta connection load as shown in Figure 27. The three phase supply voltages V_{RY}, V_{BR}, V_{YB} are connected to the three loads with a currents I_R, I_Y, I_B. The voltage between any two lines is called line voltage.

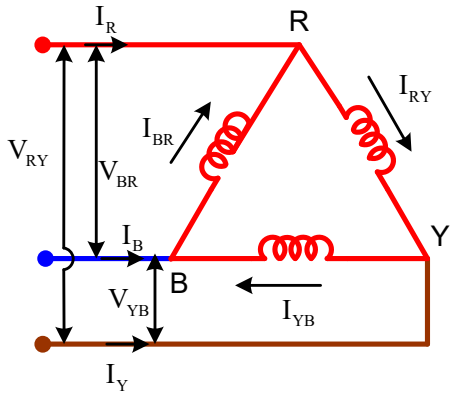


Figure 29: Delta Connection

The three line currents are:
I_R, I_Y, I_B

$$I_R = I_{RY} - I_{BR} \quad vector\ difference$$

From the phasor diagram which is as shown in Figure 30

$$I_R = \sqrt{I_{RY}^2 + I_{RB}^2 + 2I_{RY}I_{RB}cos\angle 60^\circ}$$

$$I_{RY} = I_{BR} = I_{BY} = I_P$$

$$I_R = \sqrt{I_P^2 + I_P^2 + 2I_P I_P \times 0.5} = \sqrt{3}I_P$$

Similarly

$$I_Y = \sqrt{3}I_P$$
$$I_B = \sqrt{3}I_P$$

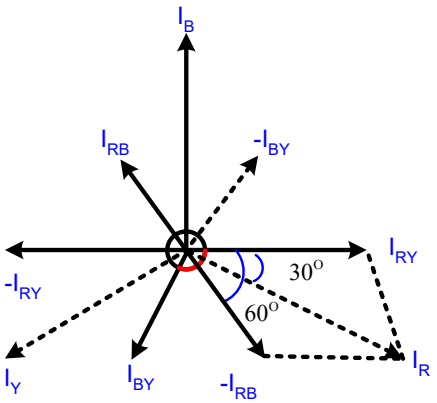


Figure 30: Phasor diagram of balanced Delta system

In delta connection the line voltage is equal to phase voltage hence Therefore

$$V_L = V_P$$

Power consumed by 3 phase circuit is

$$P = 3 \times V_P \times I_P\ cos\phi = 3V_L\frac{I_L}{\sqrt{3}} \times cos\phi = \sqrt{3}V_L I_L cos\phi$$

Power Measurement by two Wattmeter method:

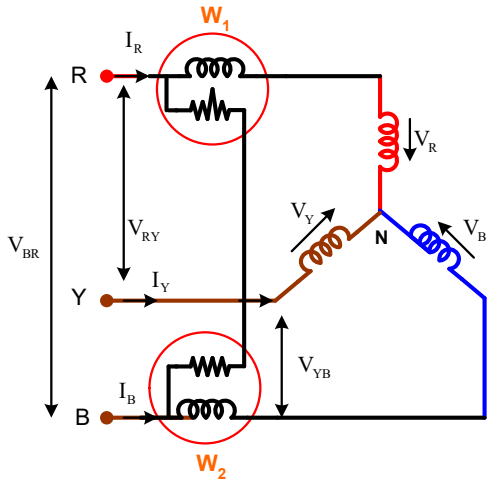


Figure 31: Measurement of Power by two wattmeter method

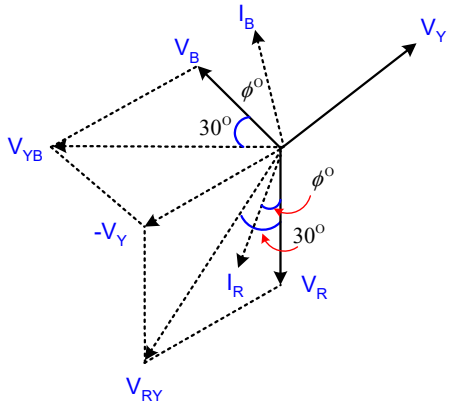


Figure 32: Phasor diagram of two wattmeter method

Reading of Wattmeter W1:

$$\begin{aligned} W_1 &= V_{RY} \times I_R \; cos(30^\circ - \phi) \\ &= V_L I_L \; cos(30^\circ - \phi) \end{aligned}$$

Reading of Wattmeter W2:

$$\begin{aligned} W_2 &= V_{YB} \times I_B \; cos(30^\circ + \phi) \\ &= V_L I_L \; cos(30^\circ + \phi) \end{aligned}$$

$$\begin{aligned} W_1 + W_2 &= V_L I_L \; cos(30^\circ - \phi) + V_L I_L \; cos(30^\circ + \phi) \\ &= V_L I_L \; [cos(30^\circ - \phi) + cos(30^\circ + \phi)] \\ &= V_L I_L \; [2cos(30^\circ) cos(\phi)] \\ &= \sqrt{3} V_L I_L \; cos\phi \end{aligned}$$

Determination of power factor.

$$\begin{aligned} W_1 - W_2 &= V_L I_L \; cos(30^\circ - \phi) - V_L I_L \; cos(30^\circ + \phi) \\ &= V_L I_L \; [cos(30^\circ - \phi) - cos(30^\circ + \phi)] \\ &= V_L I_L \; 2sin30^\circ sin\phi \\ &= V_L I_L \; sin\phi \end{aligned}$$

$$\begin{aligned} \frac{W_1 - W_2}{W_1 + W_2} &= \frac{V_L I_L \; sin\phi}{\sqrt{3} V_L I_L \; cos\phi} \\ &= \frac{tan\phi}{\sqrt{3}} \\ tan\phi &= \sqrt{3} \left[\frac{W_1 - W_2}{W_1 + W_2} \right] \\ \phi &= tan^{-1} \sqrt{3} \left[\frac{W_1 - W_2}{W_1 + W_2} \right] \\ cos\phi &= cos \left[tan^{-1} \sqrt{3} \left[\frac{W_1 - W_2}{W_1 + W_2} \right] \right] \end{aligned}$$

0.4 Problems on Three phase AC circuits

Q1) Three identical impedances are connected in delta and supplied from a 400 V 3 ϕ line. Calculate the phase and line currents and the power consumed. Each impedance is $(20 - j15)\Omega$.

Solution:

$$Z_{ph} = 20 - j15 = 25\angle -36.86^\circ$$

i) Line current
In Delta connection

$$V_{ph} = V_L = 400$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{25} = 16$$

In Delta connection line current is

$$I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 16 = 27.71$$

ii) Power consumed is

$$\begin{aligned} P_T &= \sqrt{3} V_L I_L cos\phi \\ &= \sqrt{3} 400 \times 27.71 cos \times -36.86^\circ \\ &= 15360\ W \end{aligned}$$

Q2) Three identical coils having resistance of 10 Ω and an inductance of 0.05 H each are connected in star across a 3 ϕ, 400 V 50 Hz balanced supply. Calculate the line current and the power consumed. What will be the reading of two wattmeters connected to measure the total power.

Solution:

$$\begin{aligned} R &= 10\ \Omega \\ X_L &= 2\pi f L = 2 \times \pi \times 50 \times 0.05 = 15.7 \\ Z_{ph} &= 10 + j15.7 = 18.61\angle 57.5^\circ \end{aligned}$$

$$\begin{aligned} R &= 10\ \Omega \\ cos\phi &= cos(57.5^\circ) = 0.537 \end{aligned}$$

i) Line current

$$V_{ph} = \frac{V_l}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{18.61} = 12.41A$$

In star connection line current is

$$I_L = = I_{ph} = 12.41A$$

ii) Power consumed is

$$\begin{aligned} P_T &= \sqrt{3} V_L I_L cos\phi \\ &= \sqrt{3} \times 400 \times 12.41 \times cos 57.5^\circ \\ &= 4619.65\ W \end{aligned}$$

$$W_1 + W_2 = 4619.65$$

$$\begin{aligned} cos\phi &= cos\left[tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right] \\ 0.537 &= cos\left[tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{4619.65}\right] \\ 57.5 &= tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{4619.65} \\ 1.57 &= \frac{\sqrt{3}(W_1 - W_2)}{4619.65} \\ W_1 - W_2 &= 4186.6 \end{aligned}$$

$$\begin{aligned} W_1 + W_2 &= 4619.65 \\ W_1 - W_2 &= 4186.6 \\ W_1 &= 4403W \\ W_2 &= 216.52W \end{aligned}$$

Q3) Three impedances each consisting of 20 Ω and 15 Ω inductive reactance in series are connected in star across 400 V 3 ϕ supply. Calculate the i)line current ii)phase current iii) total power consumed and iv)the p.f of the load.

Solution:

$$Z_{ph} = 20 + j15 = 25\angle 36.86^\circ$$

i) Line current
In star connection

$$V_{ph} = \frac{V_L}{\sqrt{3}} = 2\frac{400}{\sqrt{3}} = 230.94$$

ii) Phase current

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{25} = 9.23A$$

In star connection line current is

$$I_L = I_{ph} = 9.23A$$

iii) Power consumed is

$$\begin{aligned} P_T &= \sqrt{3}V_L I_L cos\phi \\ &= \sqrt{3}400 \times 9.23 cos \times 36.86^\circ \\ &= 5116\ W \end{aligned}$$

iv)the p.f

$$\begin{aligned} p.f &= cos\phi = cos \times 36.86^\circ \\ &= 0.8\ lagging \end{aligned}$$

4) Two wattmeters connected to measure the power in a three phase balanced load read $W_1 = 2000\ W$ $W_2 = 1000\ W$ Calculate i) total power and ii) power factor. When does on of the two wattmeters read the power negative

Solution:

i) Reading of each of the two wattmeters

$$Total\ power = W_1 + W_2 = 2000 + 1000 = 3000W$$

ii)Power factor

$$\begin{aligned} cos\phi &= cos\left[tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right] \\ &= cos\left[tan^{-1}\frac{\sqrt{3}(2000 - 1000)}{2000 + 1000}\right] \\ &= cos\left[tan^{-1}\frac{1732}{3000}\right] \\ &= cos\left[tan^{-1}(0.577)\right] \\ &= cos\left[30\right] = 0.866 \end{aligned}$$

The two wattmeters will read the power negative when the p.f. is less than 0.5

5) A balanced, 3 phase star connected load of 150 kW takes a leading current of 100 A, at a line voltage of 1100 V at 50 Hz. Find the circuit constants per phase.

Solution:

$$\begin{aligned} 150 \times 10^3 &= \sqrt{3}V_l I_l cos\phi \\ 150 \times 10^3 &= \sqrt{3}1100 \times 100 cos\phi \\ cos\phi &= \frac{150 \times 10^3}{\sqrt{3}1100 \times 100} \\ &= 0.7873 \\ \phi &= cos^{-1}0.7873 = 38 \end{aligned}$$

Angle is leading

$$\begin{aligned} Z_p &= \frac{V_p}{I_p} = \frac{1100}{\sqrt{3}100} = 6.35 \\ Z_p &= 6.35\angle -38 \\ &= 5 - j3.91 \\ R &= 5\ \Omega \\ X_c &= j3.91\ \Omega \\ C &= \frac{1}{2 \times \pi \times 50 \times 3.91} \\ &= 814 \times 10^{-6} F \end{aligned}$$

6) When three balanced impedances are connected in delta across a 3 phase 400 V, 50 Hz supply the line current drawn is 20 A at a lagging p.f. of 0.3. Determine the value of the impedance connected in each phase.

Solution:

$$I_p = \frac{I_l}{\sqrt{3}} = \frac{20}{\sqrt{3}} = 11.54$$

$$\begin{aligned} cos\phi &= 0.3 \\ \phi &= cos^{-1}(0.3) = 72.54 \end{aligned}$$

$$\begin{aligned} Z_p &= \frac{V_p}{I_p} = \frac{400}{11.54} = 34.64 \\ Z_p &= 34.64\angle 72.54 \\ &= 10.34 + j33.04 \end{aligned}$$

7) The power input to a 3 phase load connected to a three phase, 440 V, 50 Hz supply is measured by two wattmeter method. The readings are 40 kW and 10 kW. Calculate the power input , the p.f. and line current.

Solution:

Power input is

$$W_1 + W_2 = 40kW + 10kW = 50kW$$

ii)Power factor

$$\begin{aligned} cos\phi &= cos\left[tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right] \\ &= cos\left[tan^{-1}\frac{\sqrt{3}(40 - 10)}{40 + 10}\right] \\ &= cos\left[tan^{-1}\frac{51.96}{50}\right] \\ &= cos\left[tan^{-1}(1.039)\right] \\ &= cos\left[46.1\right] = 0.693 \end{aligned}$$

iv) Power

$$\begin{aligned} P &= \sqrt{3} \times V_L I_L cos\phi \\ 50kW &= \sqrt{3} \times 440 \times I_L \times cos(36.87^\circ) \\ I_L &= \frac{50kW}{\sqrt{3} \times 440 \times 0.693} = 94.672 \end{aligned}$$

8) Each of the two wattmeters connected to measure the input to a 3 phase circuit, reads 20 kW. What does each instruments reads, when the load p.f. is 0.866 lagging the total 3 phase power remaining unchanged.

Solution:

Power input is

$$W_1 + W_2 = 20kW$$

$$\begin{aligned} cos\phi &= cos\left[tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right] \\ 0.866 &= cos\left[tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{20}\right] \\ 30 &= \left[tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{20}\right] \\ tan(30) &= \frac{\sqrt{3}(W_1 - W_2)}{20} \\ 0.577 &= \frac{\sqrt{3}(W_1 - W_2)}{20} \\ 6.6626 &= W_1 - W_2 \end{aligned}$$

$$\begin{aligned} W_1 + W_2 &= 20kW \\ W_1 - W_2 &= 6.6626 \\ W_1 - W_2 &= 6.6626 \\ W_1 &= 13.33kW \\ W_2 &= 6.67 \end{aligned}$$

9) Two wattmeters measure the total power in a 3 phase circuit, and correctly connected. One wattmeter reads 4800 W and the other reads backwards. On reversing the connections of the later, it reads 400 Watts. What is the total power and p.f. reads 20 kW. What does each instruments reads, when the load p.f. is 0.866 lagging the total 3 phase power remaining unchanged.

Solution:

$$\begin{aligned} W_1 &= 4800W \\ W_2 &= -400W \end{aligned}$$

The total Power is

$$W_1 + W_2 = 4800W - 400 = 4400W$$

ii)Power factor

$$\begin{aligned} cos\phi &= cos\left[tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right] \\ &= cos\left[tan^{-1}\frac{\sqrt{3}(4800 + 400)}{4800 - 400}\right] \\ &= cos\left[tan^{-1}\frac{9006.6}{4400}\right] \\ &= cos\left[tan^{-1}(2.047)\right] \\ &= cos\left[63.96\right] = 0.4389 \end{aligned}$$

2020-Jan 3 c) 3 similar coils having resistance of 10 Ω and reactance of 8 Ω are connected in star across 400 V, 3 ϕ supply. Determine i) line current ii) total power iii) reading of each of the two wattmeters connected to measure power

Solution:

$$\begin{aligned} R &= 10 \\ X_L &= 8\ \Omega \\ Z_{ph} &= 10 + j8 = 12.082\angle 38.66^\circ \end{aligned}$$

i) Line current

$$\begin{aligned} V_{ph} &= \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V \\ I_{ph} &= \frac{V_{ph}}{Z_{ph}} = \frac{230.94V}{12.082\angle 38.66^\circ} \\ &= 18.0334A \\ I_L &= 18.0334A \end{aligned}$$

ii) Total power

$$\begin{aligned} P_T &= \sqrt{3} V_L I_L cos\phi \\ &= \sqrt{3}400 \times 18.0334cos \times 38.66^\circ \\ &= 9756.2116\ W \end{aligned}$$

iii) Reading of each of the two wattmeters

$$\begin{aligned} W_1 &= V_L I_L cos(30^\circ - \phi) \\ &= 400 \times 18.0334cos(30^\circ - 38.66^\circ) \\ &= 7131.1412\ W \\ W_2 &= V_L I_L cos(30^\circ + \phi) \\ &= 400 \times 18.0334cos(30^\circ + 38.66^\circ) \\ &= 2625\ W \end{aligned}$$

2020-Jan 4 b) Three coils each of impedance $20\angle 60^\circ$ are connected in star to 3 ϕ 400 V 50 Hz supply. Find the reading on each of the two wattmeters connected to measure power input.

Solution:

$$Z_{ph} = 20\angle 60^\circ$$

i) Line current

$$\begin{aligned} V_{ph} &= \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V \\ I_{ph} &= \frac{V_{ph}}{Z_{ph}} = \frac{230.94V}{20\angle 60^\circ} \\ &= 11.547A \\ I_L &= 11.547A \end{aligned}$$

iii) Reading of each of the two wattmeters

$$\begin{aligned} W_1 &= V_L I_L cos(30^\circ - \phi) \\ &= 400 \times 11.547 cos(30^\circ - 60^\circ) \\ &= 4000\ W \\ W_2 &= V_L I_L cos(30^\circ + \phi) \\ &= 400 \times 11.547 cos(30^\circ + 60^\circ) \\ &= 0\ W \end{aligned}$$

2019-June 3 c) A 3 ϕ equal impedances are connected in delta across a balanced 400 V 50 Hz 3 ϕ supply which takes a line current of 10 A at a power factor of 0.7 lagging. Calculate i) Phase current ii) Total power in W iii) Power in VA iv) Power in VAR

Solution:

i) Phase current

$$I_P = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.773$$

ii) Total power in W

$$\begin{aligned} P_T &= \sqrt{3} V_L I_L cos\phi \\ &= \sqrt{3} 400 \times 10 \times 0.7 \\ &= 4849.74\ W \end{aligned}$$

iii) Power in VA

$$\begin{aligned} P_T &= \sqrt{3} V_L I_L \\ &= \sqrt{3} 400 \times 10 \\ &= 4000\ W \end{aligned}$$

$$\begin{aligned} cos\phi &= \times 0.7 \\ \phi &= cos^{-1}(0.7) = 45.57 \end{aligned}$$

iv) Power in VAR

$$\begin{aligned} P_T &= \sqrt{3} V_L I_L sin\phi \\ &= \sqrt{3} 400 \times 10 \times sin(45.57) \\ &= 4947.47\ W \end{aligned}$$

2019-June 4 c) A balanced 3 ϕ star connected system draws power from 440 V supply. The two Wattmeters connected indicate $W_1 = 5kW$ and $W_2 = 1.2kW$. Calculate power, power factor and current in the circuit.

Solution:

$$\begin{aligned} Z &= \frac{E}{I} = \frac{377}{8\angle -30^\circ} \\ &= 47.125\angle 30^\circ \\ &= 40.8 - j23.56 \end{aligned}$$

i) the power factor

$$\begin{aligned} p.f &= cos(\phi) = cos(-30^\circ) \\ &= 0.866 \end{aligned}$$

OR

$$\begin{aligned} p.f &= \frac{R}{Z} = \frac{R}{Z} = \frac{40.8}{47.125} \\ &= 0.866 \end{aligned}$$

ii)True power Actual or True Power

$$\begin{aligned} P &= VIcos\phi = 200 \times 8 \times cos(-30^\circ) \\ &= 1385.6\ Watts \end{aligned}$$

iii) Apparent power

$$\begin{aligned} P &= VI = 200 \times 8 \\ &= 1600\ (volt\ amp) \end{aligned}$$

iv) Reactive power

$$\begin{aligned} P &= VIsin\phi = 200 \times 8 \times sin(-30^\circ) \\ &= 800\ (VAR) \end{aligned}$$

2019-Jan 3 c) Two Wattmeters are used to measure power in a 3 ϕ balanced load. The wattmeter reading are 8.2 kW and 7.5 kW. Calculate i) Total power ii) Power factor and iii) Total reactive Power

Solution:

i)Total power

$$P = W_1 + W_2 = 8.2 - 7.5 = 0.7kW$$

ii)Power factor

$$\begin{aligned} cos\phi &= cos\left[tan^{-1}\frac{\sqrt{3}(W_1-W_2)}{W_1+W_2}\right] \\ &= cos\left[tan^{-1}\frac{\sqrt{3}(8.2+7.5)}{8.2-7.5}\right] \\ &= cos\left[tan^{-1}\frac{27.19}{8.2-7.5}\right] \\ &= cos\left[tan^{-1}\frac{27.19}{0.7}\right] \\ &= cos\,[88.52] = 0.0257 \end{aligned}$$

$$\phi = cos^{-1}(0.0257) = 88.527$$

iv) Reactive power

$$\begin{aligned} P &= 700 \times sin\phi = 700 \times sin(88.527^\circ) \\ &= 699.76\ (VAR) \end{aligned}$$

2019-Jan 4 b) Three coils are connected in delta to a three phase, three wire, 400 V 50 Hz supply and take a line current of 5 A at 0.8 p.f lagging. Calculate the resistance and inductance of the coils.

Solution:

$$Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{400}{5} = 80$$

$$\begin{aligned} cos\phi &= 0.8 \\ \phi &= cos^{-1}(0.8) = 36.87 \end{aligned}$$

$$\begin{aligned} Z_{Ph} &= 80\angle 36.87 = 64 + j48 \\ R &= 64 \\ X_L &= 48 \\ L &= \frac{1}{2 \times f \times X_L 48} \\ &= \frac{1}{2 \times 50 \times 48} = 0.631 \times^{-6} H \end{aligned}$$

2020-Jan (2017-scheme) 7 b) A balanced delta connected of (8+j6)Ω per phase is connected to a 3 ϕ 230 Volts 50 Hz AC supply Find i) Phase current ii) Line current iii) Power factor iv) Power v) Total reactive Power vi) Volt-Amp

Solution:

$$Z_P = 8 + j6 = 10\angle 36.87^\circ$$

i) the power factor

$$\begin{aligned} p.f &= cos(\phi) = cos(36.87^\circ) \\ &= 0.8 \end{aligned}$$

i) Phase current

$$V_P = 230V$$

$$\begin{aligned} I_P &= \frac{230}{10} = 23A \\ I_L &= = \sqrt{3}\frac{230}{10} = 39.84A \end{aligned}$$

iv) Power

$$\begin{aligned} P &= \sqrt{3} \times V_P I_P cos\phi \\ &= \sqrt{3} \times 230 \times 39.84 \times cos(36.87^\circ) \\ &= 12696.8\ Watts \end{aligned}$$

v) Reactive power

$$\begin{aligned} P &= \sqrt{3} \times V_P I_P sin\phi = \sqrt{3} \times 230 \times 39.84 \times sin(36.87^\circ) \\ &= 9522.7\ (VAR) \end{aligned}$$

vi) Volt-Amp

$$\begin{aligned} P &= \sqrt{3} \times V_P I_P = \sqrt{3} \times 230 \times 39.84 \\ &= 15871\ (V-A)(volt\ amp) \end{aligned}$$

2020-Jan (2017-scheme) 8 b) The power input to a 3 ϕ induction motor running on 400 V, 50Hz AC supply. The wattmeter reading were 3000 W and -1000 W. Calculate i) Total input power ii) Power factor iii) Line current

Solution:

i)Total input power

$$P = W_1 + W_2 = 3000 + 1000 = 4000$$

ii)Power factor

$$\begin{aligned} cos\phi &= cos\left[tan^{-1}\frac{\sqrt{3}(W_1-W_2)}{W_1+W_2}\right] \\ &= cos\left[tan^{-1}\frac{\sqrt{3}(3000-1000)}{3000+1000}\right] \\ &= cos\left[tan^{-1}\frac{3464.1}{4000}\right] \\ &= cos\,[40.89] = 0.756 \end{aligned}$$

$$\phi = cos^{-1}(0.756) = 40.89$$

2020-Jan (2017-scheme) 7 b) A balanced star connected load of (8+j6)Ω per phase is connected to a 3 ϕ 230 Volts 50 Hz AC supply Find i) Line

current ii) Power factor iii) Power iv) Total reactive Power vi) Total Volt-Amp

Solution:

$$Z_P = 8 + j6 = 10\angle 36.87^\circ$$

i) the power factor

$$p.f = cos(\phi) = cos(36.87^\circ) = 0.8$$

i) Phase current

$$V_P = \frac{230}{\sqrt{3}} = 132.8V$$

$$I_P = I_L = \frac{132.8}{10} = 13.28A$$

iv) Power

$$P = \sqrt{3} \times V_L I_L cos\phi = \sqrt{3} \times 230 \times 13.28 \times cos(36.87^\circ) = 4232.3\ Watts$$

v) Reactive power

$$P = \sqrt{3} \times V_L I_L sin\phi = \sqrt{3} \times 230 \times 13.28 \times sin(36.87^\circ) = 3174\ (VAR)$$

vi) Volt-Amp

$$P = \sqrt{3} \times V_L I_L \phi = \sqrt{3} \times 230 \times 13.28 = 5290\ (V-A)(volt\ amp)$$

2018-Jan (2015-scheme) 7 c) A 3 ϕ star connected system has 4 Ω resistance in series with an inductance of 10 mH per phase is applied voltage is 415 V with 50 Hz Find the Power drawn by the circuit.

Solution:

$$X_L = 2\pi fL = 2 \times \pi \times 50 \times 10 \times 10^{-3} = 3.14$$

$$Z_P = 4 + j3.14 = 5\angle 38.13^\circ$$

i) the power factor

$$p.f = cos(\phi) = cos(38.13^\circ) = 0.7866$$

i) Phase current

$$V_P = \frac{415}{\sqrt{3}} = 239.6V$$

$$I_P = I_L = \frac{239.6}{5} = 48A$$

iv) Power

$$P = \sqrt{3} \times V_L I_L cos\phi = \sqrt{3} \times 415 \times 48 \times cos(38.13^\circ) = 27140\ Watts$$

2019-June (2015-scheme) 7 c) Three identical resistors are connected in star across 400 V, 50 Hz AC supply. The line current is 10 Amps. Find the Power power consumed when resistors are connected in delta with line current remaining the same.

Solution:

Star connected i) Phase current

$$V_P = \frac{400}{\sqrt{3}} = 230.9V$$

$$I_P = I_L = 10A$$

$$Z_P = \frac{230.9}{10} = 23.09A$$

iv) Power

$$P = \sqrt{3} \times V_L I_L = \sqrt{3} \times 415 \times 10 = 7188\ Watts$$

Delta connected

i) Phase current

$$I_P = \frac{400}{23.09} = 17.32A$$

$$I_L = = \sqrt{3} \times 17.32 = 30A$$

Power

$$P = \sqrt{3} \times V_L I_L = \sqrt{3} \times 415 \times 30 = 21564\ Watts$$

2020-Jan (2017-scheme) 8 b) A balanced three phase star connected load draws power form 440 V supply. The two wattmeters connected indicate W_1 = 750 W and W_2 = 1.5 kW. Calculate i) Power ii) Power factor iii) current in the circuit. If the W_1 = wattmeter is reversed, what would be the phase angle between voltage and current

Solution:

i)Total input power

$$P = W_1 + W_2 = 1500 + 750 = 2250$$

ii)Power factor

$$\begin{aligned} cos\phi &= cos\left[tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right] \\ &= cos\left[tan^{-1}\frac{\sqrt{3}(1500 - 750)}{1500 + 750}\right] \\ &= cos\left[tan^{-1}\frac{1299}{2250}\right] \\ &= cos\left[tan^{-1}(0.577)\right] \\ &= cos\left[30\right] = 0.866 \end{aligned}$$

$$\phi = cos^{-1}(0.756) = 40.89$$