****Series R-L-C Circuits****

1. A series RLC circuit has R = 20 Ω, L = 0.1 H and C = 100 μF. It is connected to a 230 V, 50 Hz supply. Calculate impedance, current and power factor.

Solution:

$X_L = 2\pi fL = 31.42\ \Omega$

$X_C = 1/(2\pi fC) = 31.83\ \Omega$

$X = X_L - X_C = -0.41\ \Omega$

$Z = \sqrt{(R^2 + X^2)} = 20.004\ \Omega \approx 20\ \Omega$

$I = V/Z = 11.5\ A$

$PF = R/Z \approx 1$ (almost unity, slightly leading)

2. A series RLC circuit has R = 40 Ω, L = 0.2 H, C = 50 μF and is supplied by 200 V, 50 Hz. Find the current, voltage across each element and power factor.

Solution:

$X_L = 62.83\ \Omega$

$X_C = 63.66\ \Omega$

$X = X_L - X_C = -0.83\ \Omega$

$Z = \sqrt{(40^2 + (-0.83)^2)} = 40.01\ \Omega$

$I = 200/40.01 = 5\ A$

$V_R = 200\ V$, $V_L = 314.15\ V$, $V_C = 318.3\ V$

$PF = 40/40.01 \approx 1$ (leading)

3. In a series RLC circuit, R = 15 Ω, L = 0.25 H, C = 80 μF, V = 150 V, f = 50 Hz. Calculate impedance, current, active power and reactive power.

Solution:

$X_L = 78.54\ \Omega$

$X_C = 39.79\ \Omega$

$X = 38.75\ \Omega$

$Z = \sqrt{(15^2 + 38.75^2)} = 41.55\ \Omega$

$I = 3.61\ A$

$P = I^2R = 195.5\ W$

$Q = I^2X = 504.5\ VAR$ (inductive)

4. A series RLC circuit takes 5 A current at 0.8 power factor lagging when connected to 200 V, 50 Hz supply. If R = 32 Ω, find L and C.

Solution:

$Z = V/I = 40\ \Omega$

$\cos\theta = 0.8 \rightarrow \sin\theta = 0.6$

$X = Z \sin\theta = 24\ \Omega$ (inductive)

$X_L - X_C = 24$

Also $R = 32\ \Omega$ (given)

We need one more condition. Assume the circuit is at a frequency where additional data is needed; typically we solve with given data.

(Note: Complete data usually specifies more. For this: $X = 24\ \Omega$)

5. A 230 V, 50 Hz supply is connected to a series combination of R = 25 Ω, L = 0.15 H and C = 120 μF. Find the total impedance, current and power consumed.

Solution:

$X_L = 47.12\ \Omega$

$X_C = 26.53\ \Omega$

$X = 20.59\ \Omega$

$Z = \sqrt{(25^2 + 20.59^2)} = 32.39\ \Omega$

$I = 7.1\ A$

$P = I^2R = 1260\ W$

Series Resonance

6. A series RLC circuit has R = 10 Ω, L = 0.2 H and C = 50 μF. Find the resonant frequency, Q-factor and bandwidth.

Solution:

Resonant frequency $f_0 = 1/(2\pi\sqrt{(LC)}) = 1/(2\pi\sqrt{(0.2\times50\times10^{-6})}) = 50.33\ Hz$

$Q = (1/R)\sqrt{(L/C)} = (1/10)\sqrt{(0.2/50\times10^{-6})} = 20$

Bandwidth $= f_0/Q = 2.52\ Hz$

7. A series resonant circuit has L = 0.1 H, C = 100 μF and R = 5 Ω. It is connected to a 50 V variable frequency supply. Find resonant frequency, current at resonance and voltage across L and C at resonance.

Solution:

$f_0 = 1/(2\pi\sqrt{(LC)}) = 50.33\ Hz$

At resonance $Z = R = 5\ \Omega$

$I_0 = V/R = 10\ A$

$V_L = V_C = I_0 \times X_L = I_0 \times (2\pi f_0 L) = 316.2\ V$

8. In a series RLC circuit, resonance occurs at 1 kHz. L = 50 mH and R = 20 Ω. Find the value of C and the Q-factor.

Solution:

$f_0 = 1000 = 1/(2\pi\sqrt{(LC)})$

$\sqrt{(LC)} = 1/(2\pi \times 1000)$

$LC = 2.533 \times 10^{-8}$

$C = 2.533 \times 10^{-8} / 0.05 = 0.5066\ \mu F$

$Q = (2\pi f_0 L)/R = (2\pi \times 1000 \times 0.05)/20 = 15.7$

9. A series RLC circuit is resonant at 50 Hz. R = 10 Ω, L = 0.2 H. Find C and the current when the circuit is connected to 100 V, 50 Hz supply.

Solution:

At resonance $X_L = X_C$

$X_L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$

$C = 1/(2\pi f X_L) = 50.66\ \mu F$

$I = V/R = 10\ A$

10. Explain Series Resonance and calculate the resonant frequency for R = 8 Ω, L = 0.4 H, C = 40 μF.

Solution:

Series Resonance occurs when X_L = X_C, so impedance is minimum (Z = R) and current is maximum. The circuit behaves as purely resistive and power factor is unity.

$f_0 = 1/(2\pi\sqrt{(LC)}) = 1/(2\pi\sqrt{(0.4\times40\times10^{-6})}) = 39.79$ Hz

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Parallel R-L-C Circuits

11. A parallel RLC circuit has R = 40 Ω, L = 0.1 H and C = 200 μF connected across 230 V, 50 Hz supply. Find the total current and power factor.

Solution:

I_R = 230/40 = 5.75 A

X_L = 31.42 Ω → I_L = 7.32 A

X_C = 15.92 Ω → I_C = 14.45 A

Net reactive current I_X = I_C – I_L = 7.13 A (capacitive)

$I = \sqrt{(I_R^2 + I_X^2)} = 9.16$ A

PF = I_R/I = 0.628 leading

12. In a parallel circuit, a 30 Ω resistor, 0.15 H inductor and 100 μF capacitor are connected across 100 V, 50 Hz. Calculate branch currents and total current.

Solution:

I_R = 3.333 A

X_L = 47.12 Ω → I_L = 2.122 A

X_C = 31.83 Ω → I_C = 3.141 A

I_X = I_C – I_L = 1.019 A

$I = \sqrt{(3.333^2 + 1.019^2)} = 3.485$ A

13. A parallel RLC circuit draws 10 A at 0.6 power factor leading from 200 V supply. The resistor takes 6 A. Find the currents in inductive and capacitive branches (assume pure L and C).

Solution:

$I_R = 6$ A

$\cos\theta = 0.6 \rightarrow I_X = I \sin\theta = 8$ A (leading, so capacitive)

Net $I_C - I_L = 8$ A

(Additional data needed for individual values; net reactive current is 8 A capacitive)

Parallel Resonance (Anti-resonance)

14. A parallel resonant circuit has L = 0.2 H, C = 50 μF and R = 10 Ω (resistance in inductive branch). Find the resonant frequency.

Solution:

For parallel resonance (practical case):

$f_0 = (1/(2\pi\sqrt{(LC)})) \times \sqrt{(1 - R^2C/L)}$

$= 50.33 \times \sqrt{(1 - (100 \times 50 \times 10^{-6})/0.2)} = 50.33 \times \sqrt{(1 - 0.025)} = 49.7$ Hz

15. Explain Parallel Resonance and find the resonant frequency for a parallel circuit with L = 0.1 H, C = 100 μF (ideal case, R = 0).

Solution:

Parallel Resonance (anti-resonance) occurs when the inductive and capacitive susceptances cancel each other. The total impedance becomes maximum and the line current becomes minimum. Power factor is unity.

Ideal resonant frequency $f_0 = 1/(2\pi\sqrt{(LC)}) = 50.33$ Hz

16. A parallel combination of 0.25 H inductor and 40 μF capacitor is connected to a 50 V variable frequency source. Find the resonant frequency and dynamic impedance (assume coil resistance = 5 Ω).

Solution:

$f_0 \approx 1/(2\pi\sqrt{(LC)}) = 50.33$ Hz (approximate)

Dynamic impedance $Z_{dyn} = L/(R\,C) = 0.25/(5 \times 40\times10^{-6}) = 1250$ Ω

Additional Mixed Problems

17. A series RLC circuit has R = 12 Ω, L = 0.15 H, C = 100 μF. Supply = 120 V, 50 Hz. Calculate Z, I, PF and power.

Solution:

$X_L = 47.12$ Ω

$X_C = 31.83$ Ω

$X = 15.29$ Ω

$Z = 19.45$ Ω

$I = 6.17$ A

PF = 0.617 lagging

P = 456.5 W

18. Series RLC circuit: R = 25 Ω, X_L = 60 Ω, X_C = 40 Ω, V = 200 V. Find current, PF and voltages across L and C.

Solution:

X = 20 Ω

Z = 32.02 Ω

I = 6.246 A

PF = 0.781 lagging

V_L = 374.8 V, V_C = 249.8 V

19. A series resonant circuit has bandwidth of 200 Hz and Q-factor of 25. Find the resonant frequency and half-power frequencies.

Solution:

Bandwidth = $f_0/Q \Rightarrow f_0 = 200 \times 25 = 5000$ Hz

Half-power frequencies: $f_1 = f_0 – BW/2 = 4900$ Hz

$f_2 = f_0 + BW/2 = 5100$ Hz

20. A parallel RLC circuit with R = 50 Ω, L = 0.2 H, C = 50 μF is connected to 100 V, 50 Hz. Find total current and power factor.

Solution:

I_R = 2 A

X_L = 62.83 Ω → I_L = 1.591 A

$X_C = 63.66\ \Omega \rightarrow I_C = 1.571$ A

$I_X \approx 0.02$ A (very small)

$I \approx 2$ A

PF ≈ 1

21. In a series RLC circuit at resonance, current is 10 A. R = 5 Ω. If the voltage across capacitor is 500 V, find the value of L and C at f = 50 Hz.

Solution:

$V = I R = 50$ V (supply voltage at resonance)

$V_C = I X_C = 500 \Rightarrow X_C = 50\ \Omega$

$C = 1/(2\pi f X_C) = 63.66\ \mu F$

$X_L = X_C = 50\ \Omega \Rightarrow L = 0.159$ H

22. A series RLC circuit has R = 10 Ω, L = 0.1 H, C = 200 μF. Find the frequency at which the circuit has unity power factor and the impedance at that frequency.

Solution:

Unity PF occurs at resonance.

$f_0 = 1/(2\pi\sqrt{(LC)}) = 35.59$ Hz

At resonance $Z = R = 10\ \Omega$
