**Proof of Capacitor Current in a Sinusoidal AC Supply**

The fundamental relationship for a capacitor is:

$$i_C = C\frac{dv_C}{dt}$$

Consider a pure (lossless) capacitor connected to a sinusoidal voltage source:

$$v_C(t) = V_m \sin(\omega t)$$

Differentiate with respect to time:

$$\frac{dv_C}{dt} = V_m \omega \cos(\omega t)$$

Therefore the current is:

$$i_C(t) = C \cdot V_m \omega \cos(\omega t) = \omega C V_m \cos(\omega t)$$

We can rewrite this as:

$$i_C(t) = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

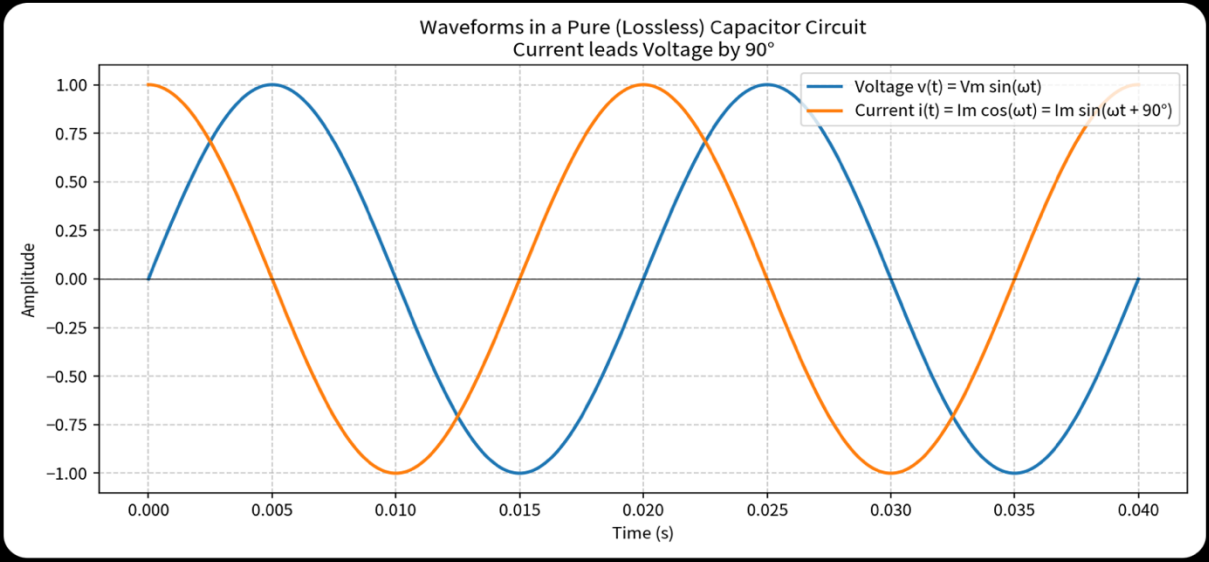
where $I_m = \omega C V_m = \dfrac{V_m}{X_C}$ and $X_C = \dfrac{1}{\omega C}$.

**Conclusion:**
In a pure capacitor, the current **leads** the voltage by **90°** (or $\pi/2$ radians).

**Waveforms: Pure (Lossless) Capacitor Circuit**

- Voltage (blue): $v(t) = V_m \sin(\omega t)$
- Current (orange): $i(t) = I_m \cos(\omega t) = I_m \sin(\omega t + 90^\circ)$

Current reaches its positive peak exactly when voltage is zero and rising, confirming the 90° phase lead.



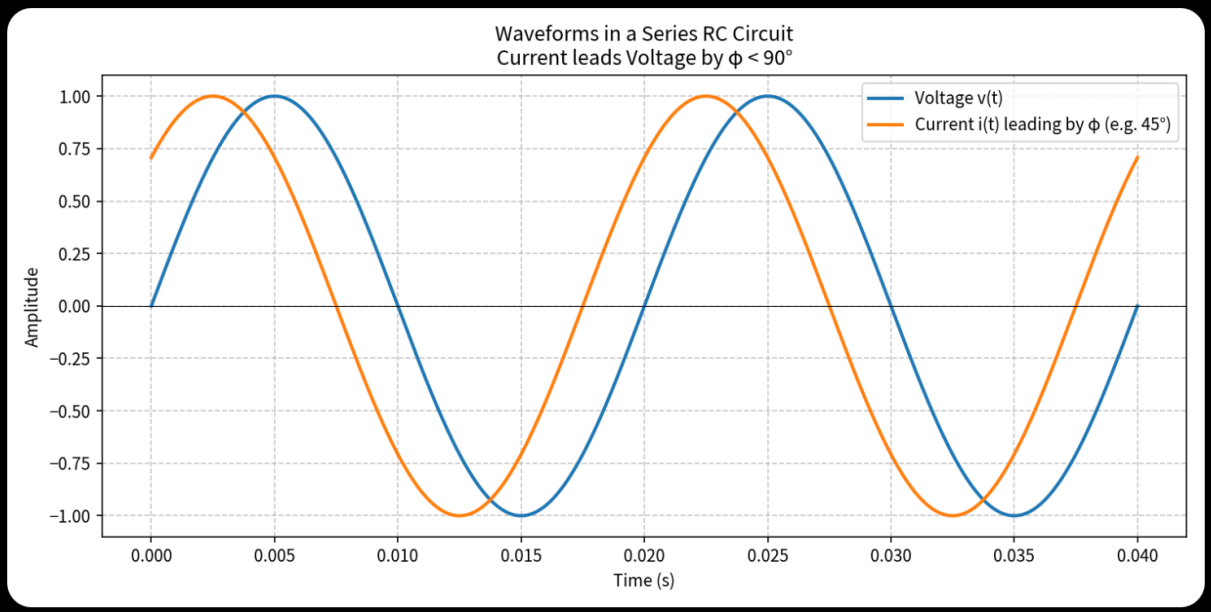

**Waveforms: Series RC Circuit**

In a series RC circuit the phase difference is less than 90°.

The current still **leads** the voltage, but by an angle $\varphi$ where:

$$\varphi = \tan^{-1}\left(\frac{X_C}{R}\right) \quad (0^\circ < \varphi < 90^\circ)$$

- Voltage (blue): $v(t)$
- Current (orange): leads voltage by $\varphi$ (example shown for $\varphi = 45^\circ$)



**Key Points Summary**

| Circuit | Phase Relationship | Power Factor | Current Expression |
|---|---|---|---|
| Pure Capacitor | Current leads Voltage by 90° | Zero (leading) | $i = \omega C V_m \cos(\omega t)$ |
| Series RC | Current leads Voltage by $\varphi < 90^\circ$ | Leading | $i = I_m \sin(\omega t + \varphi)$ |